acteristics. However, the discovery of these defects enables one to concentrate attention on the zones which need particular care during tests and debugging the design.

The reported results show that the preliminary flow visualization in the models of laser pumping loops considerably simplifies the work and makes it possible to avoid mistakes at the stage of developing and designing a laser. The design procedure is very simple and the consumed time and capital outlays are incomparably lower than those related with adapting a loop from the unsuccessfully designed one. In modernizing the available lasers, hydraulic modeling is an effective tool for obtaining the desired result with minimally changing the design.

NOTATION

D, inner diameter of the vane array of the rotor; L, length of the vane array along the axis of rotation; U, peripheral velocity of the rotor.

LITERATURE CITED

1. F. E. Kassadi, Aérodinamicheskaya Tekhnika, No. 7, 135-148 (1986).
2. T. P. Balmer, Design News, No. 9, 20-23 (1966).
3. A. G. Korovkin, Prom. Aerodin., No. 1(33), 71-80, Moscow (1986).
4. S. M. Grigor'ev and G. A. Gridneva, Zap. LSKhI, **149**, No. 1, 76-86 (1970).
5. B. Eck, Fans, Pergamon Press, Oxford (1973), pp. 156-181.
6. K. Yamafuji, Bull. JSME, **18**, No. 123, 1018-1024 (1970).
7. A. M. Porter and E. A. Markland, J. Mech. Eng. Sci., **12**, No. 6, 421-431 (1970).
8. N. N. Suntsov, The Analogy Methods in Aerohydrodynamics [in Russian], Moscow (1986).
9. D. J. Allen, Conf. on Fan Design and Application, Guildford. England, Sept. 7-9 (1982), pp. 355-386.

THE KINETIC MODEL OF PARTICLE TRANSFER IN TURBULENT FLOWS WITH CONSIDERATION OF COLLISIONS

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The work presents the kinetic model of particle dynamics in turbulent flows taking into consideration inelastic collisions. Transfer coefficients of the dispersed phase in constrained flows are found on the basis of this model.

To describe a particle movement in rarefied dispersed flows (i.e. for low volume concentration of the dispersed phase), greater attention should be paid to the interaction between particles and turbulent pulsations of the carrier flow, since the role of collisions between the particles proper is not essential. The kinetic equation for the probability density function [PDF] of the particle velocity in turbulent flows without taking account of collisions was obtained in [1, 2]. For large particles ($\tau/T >> 1L$) in an isotropic turbulent flow this equation develops into the known Fokker–Planck equation for the Brownian movement [3, 4]. A solution of the equation for the PDF can be constructed with the help of the perturbation method [4-6] widely used in the kinetic theory of gases for the solution of the Boltzmann equation [7, 8]. On the contrary, in the case of the analysis of particle dynamics in sufficiently dense dispersed flows the collisions of particles between themselves play a determining role. An elementary kinetic theory of highly concentrated dispersed systems is formed in [9]. Studies [10, 11] offer the kinetic models of particle transfer in dispersed flows, based on the solution of the Boltzmann equation by the perturbation method and further developing

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the Enskog approach for dense gases for the case of inelastic particle collisions. The present work proposes a kinetic model for describing particles in turbulent flows taking into consideration their interaction during collisions, which generalizes the results for rarefied [4, 6] and concentrated [10, 11] dispersed systems.

The equation for the PDF of the velocity of large particles disregarding their rotation can be presented in the form

$$T[F]+J[F,\ F]=N[F],$$
$$T[F]=\frac{D}{\tau^2}\frac{\partial^2 F}{\partial v_k \partial v_k}+\frac{1}{\tau}\frac{\partial (v_k-V_k)F}{\partial v_k}, \tag{1}$$
$$N[F]=\frac{\partial F}{\partial t}+v_k\frac{\partial F}{\partial x_k}+\left(\frac{U_k-V_k}{\tau}+f_k\right)\frac{\partial F}{\partial v_k}.$$

Here T[F] is the operator, describing the particle interaction with the carrier turbulent flow; J[F, F] is the Boltzmann operator of collisions in the Enskog form [7], and N[F] is the convective operator, defining the deviation of the probability density of the particle velocity from the equilibrium Maxwellian distribution.

The integration of Eq. (1) in the velocity space can result in equations for the concentration, mean velocity, and pulsation energy of the dispersed phase. The equation for the volume concentration of particles has the form

$$\frac{\partial \varphi}{\partial t}+\frac{\partial \varphi V_k}{\partial x_k}=0. \tag{2}$$
$$\varphi=\int F d\mathbf{v},\quad V_i=\int v_i F d\mathbf{v}/\varphi.$$

Let us write the equation for the dispersed phase movement:

$$\frac{\partial V_i}{\partial t}+V_k\frac{\partial V_i}{\partial x_k}=\frac{U_i-V_i}{\tau}+f_i-\frac{1}{\varphi}\frac{\partial P_{ik}}{\partial x_k}. \tag{3}$$

Here $P_{ij} = P_{ij}^k + P_{ij}^c$ is the total stress tensor in the dispersed phase; $P_{ij}^k = \varphi\langle v_i'v_j'\rangle$ are the stresses caused by the kinetic pulsation movement of particles, and P_{ij}^c are the stresses, which define the momentum transfer during particle collisions.

The equation of the balance of the particle kinetic pulsation energy has the form:

$$\frac{\partial \varphi k_p}{\partial t}+\frac{\partial \varphi V_k k_p}{\partial x_k}=P_{ik}\frac{\partial V_i}{\partial x_k}+\frac{\varphi}{\tau}\left(3\frac{D}{\tau}-2k_p\right)-\frac{\partial q_k}{\partial x_k}-Q. \tag{4}$$

The first term on the right side of Eq. (4) describes the generation of the dispersed phase pulsation energy from the mean movement due to the velocity shift. The second term defines the pulsation energy exchange with the turbulent carrier flow. The third term describes the pulsation energy diffusion transfer. The quantity Q determines the pulsation energy dissipation due to particle collisions. The total pulsation energy flux $q_i = q_i^k + q_i^c$ is combined from the kinetic part $q_i^k = \varphi\langle v_i'v_k'v_k'\rangle/2$ and from the term q_i^c, stipulated by particle collisions.

It should be noted that the division of the stress tensor and pulsation energy flux into the kinetic part and the part stipulated by collisions is arbitrary and meaningful from the procedural (computational) point of view.

In accordance with the solution of the kinetic equation for dense gases by the Enskog method [7], the integral of collisions is presented in the form

$$J[F,\ F]=YJ_0[F,\ F]+J_1[F,\ F]+\ldots, \tag{5}$$

where $J_0[F, F]$ is the Boltzmann collision integral ($\varphi \to 0$); $J_1[F, F]$ is the contribution of the first terms into the expansion of the collision integral into the Taylor series in terms of the parameter, proportional to the particle dimension, and $Y(\varphi)$ is the quantity which can be interpreted as the ratio of the number of particle collisions in concentrated and rarefied media.

Within the framework of the assumption of a small deviation of particle distribution from equilibrium , the solution of Eq. (1) with allowance for (5) can be found in the form of a series $F = F_0 + F_1+ \ldots$, where the functions F_0 and F_1 satisfy the equations

$$T[F_0]+YJ_0[F_0,\ F_0]=0, \tag{6}$$

$$T[F_1]+Y(J_0[F_0,\ F_1]+J_0[F_1,\ F_0])=N[F_0]-J_1[F_0,\ F_0]=L[F_0]. \tag{7}$$

The Maxwellian distribution is the solution for Eq. (6)

$$F_0 = \varphi\left(\frac{3}{4\pi k_p}\right)^{3/2} \exp\left(-\frac{3v_k' v_k'}{4k_p}\right). \tag{8}$$

According to (8):

$$\langle v_i' v_j' \rangle = \frac{2}{3} k_p \delta_{ij}, \quad \langle v_i' v_k' v_k' \rangle = 0. \tag{9}$$

The quantities P_{ij}^C, q_i^C, and Q, taking into account possible energy loss because of inelastic particle collisions, have the form [10, 11]:

$$P_{ij}^C = \frac{4}{5}(1+e)\varphi^2 Y\left(\langle v_i' v_j' \rangle + \frac{1}{2}\langle v_k' v_k' \rangle \delta_{ij}\right) - \\ -\frac{4}{3}(1+e)\varphi^2 Y d_p \sqrt{\frac{2k_p}{3\pi}}\left(\frac{3}{5}S_{ij} + \frac{\partial V_k}{\partial x_k}\delta_{ij}\right), \tag{10}$$

$$q_i^C = \frac{3}{5}(1+e)\varphi^2 Y \langle v_i' v_k' v_k' \rangle - 2(1+e)\varphi^2 Y d_p \sqrt{\frac{2k_p}{3\pi}} \frac{\partial k_p}{\partial x_i}, \tag{11}$$

$$Q = 2(1-e^2)\varphi^2 Y k_p \left[\frac{4}{d_p}\left(\frac{2k_p}{3\pi}\right)^{1/2} - \frac{\partial V_k}{\partial x_k}\right], \tag{12}$$

where

$$S_{ij} = \frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} - \frac{2}{3}\frac{\partial V_k}{\partial x_k}\delta_{ij}.$$

Substituting Eq. (8) into the right side of Eq. (7) and using (2)-(4) taking into account (9)-(12), we obtain

$$L[F_0] = F_0\left\{\left[1 + \frac{2}{5}(1+e)(3e-1)\varphi Y\right]\frac{3}{2k_p}\left(v_i' v_j' - \frac{v_k' v_k'}{3}\delta_{ij}\right)\frac{\partial V_i}{\partial x_j} + \right. \\ \left. + \left[1 + \frac{3}{5}(1+e)^2(2e-1)\varphi Y\right]\left(\frac{3v_k' v_k'}{4k_p} - \frac{5}{2}\right)\frac{v_i'}{k_p}\frac{\partial k_p}{\partial x_i}\right\}. \tag{13}$$

Expression (13) is written ignoring the term with the dispersed phase concentration gradient, contributing noticeably into the diffusive transfer of the pulsation energy only for essentially inelastic particle collisions.

Let us define separately the solutions to equations

$$T[F_{1T}] = L[F_0], \tag{14}$$

$$J_0[F_0, F_{1J}] + J_0[F_{1J}, F_0] = \frac{1}{Y} L[F_0], \tag{15}$$

satisfying the normalization conditions

$$\int F_1 d\mathbf{v} = \int v_i' F_1 d\mathbf{v} = \int v_k' v_k' F_1 d\mathbf{v} = 0.$$

Then, if the solution to Eqs. (14) and (15) has the form $F_{1T} = A_T\varphi(v_i)$, $E_{1J} = A_J\varphi(v_i)$, where A_T and A_J are constants, then the solution to Eq. (7) will be

$$F_1 = \frac{A_T A_J}{A_T + A_J}\varphi(v_i). \tag{16}$$

The solution to Eq. (14) taking account of (13) has the form

$$F_{1T} = -F_0\frac{\tau}{k_p}\left\{\frac{3}{4}\left[1 + \frac{2}{5}(1+e)(3e-1)\varphi Y\right] \times \right. \\ \times \left(v_i' v_j' - \frac{v_k' v_k'}{3}\delta_{ij}\right)\frac{\partial V_i}{\partial x_j} + \left[1 + \frac{3}{5}(1+e)^2(2e-1)\varphi Y\right] \times \tag{17}$$

$$\times\left(\frac{3v_k' v_k'}{4k_p}-\frac{5}{2}\right)\frac{v_i'}{3}\frac{\partial k_p}{\partial x_i}\Big\}.$$

The approximate solution to Eq. (15), taking into account (13), obtained by the retention of only one term in the expansion in terms of the Sonin polynomials and conforming to the thirteen-moment Grad approximation in the case of inelastic interaction of solid spheres, has the form

$$F_{1J}=-F_0\left(\frac{3\pi}{2}\right)^{1/2}\frac{5d_p}{64k_p^{3/2}\varphi Y}\Big\{\frac{4}{(1+e)(3-e)}\times$$
$$\times\left[1+\frac{2}{5}(1+e)(3e-1)\varphi Y\right]\left(v_i'v_j'-\frac{v_k'v_k'}{3}\delta_{ij}\right)\frac{\partial V_i}{\partial x_j}+$$
$$+\frac{32}{(1+e)(49-33e)}\left[1+\frac{3}{5}(1+e)^2(2e-1)\varphi Y\right]\times \tag{18}$$
$$\times\left(\frac{3v_k'v_k'}{4k_p}-\frac{5}{2}\right)v_i'\frac{\partial k_p}{\partial x_i}\Big\}.$$

Let us define the kinetic stresses and the pulsation energy flux

$$P_{ij}^K=\int v_i'v_j'(F_0+F_1)\,dv=\frac{2}{3}\varphi k_p\delta_{ij}-\varphi\nu_K S_{ij}, \tag{19}$$

$$q_i^K=\frac{1}{2}\int v_i'v_k'v_k'F_1 dv=-\varphi\Lambda_K\frac{\partial k_p}{\partial x_i}, \tag{20}$$

where, according to Eqs. (16)-(18), the viscosity and pulsation energy diffusion coefficients ν_k and Λ_k, resulting from the kinetic transfer, are equal to:

$$\nu_K=\frac{\nu_T\nu_J}{\nu_T+\nu_J},\quad \Lambda_K=\frac{\Lambda_T\Lambda_J}{\Lambda_T+\Lambda_J},$$
$$\nu_T=\left[1+\frac{2}{5}(1+e)(3e-1)\varphi Y\right]\frac{\tau k_p}{3},$$
$$\nu_J=\sqrt{\frac{2\pi k_p}{3}}\frac{5d_p}{24(1+e)(3-e)\varphi Y}\left[1+\frac{2}{5}(1+e)(3e-1)\varphi Y\right], \tag{21}$$
$$\Lambda_T=\frac{10}{27}\left[1+\frac{3}{5}(1+e)^2(2e-1)\varphi Y\right]\tau k_p,$$
$$\Lambda_J=\sqrt{\frac{2\pi k_p}{3}}\frac{25d_p}{6(1+e)(49-33e)\varphi Y}\left[1+\frac{3}{5}(1+e)^2(2e-1)\varphi Y\right].$$

The stress and the diffusion flux of the pulsation energy, stipulated by the momentum and energy transfer as a result of particle collisions, according to Eqs. (10), (11), (19), and (20), are defined by relations

$$P_{ij}^C=\left[\frac{4}{3}(1+e)\varphi^2 Yk_p-\varphi\xi\frac{\partial V_k}{\partial x_k}\right]\delta_{ij}-\varphi\nu_C S_{ij}, \tag{22}$$

$$q_i^C=-\varphi\Lambda_C\frac{\partial k_p}{\partial x_i}. \tag{23}$$

Here the shear viscosity coefficient ν_c, the volume viscosity coefficient ξ, and the pulsation energy diffusion coefficient Λ_c, caused by collisions, are equal to:

$$\nu_C=\frac{4}{5}(1+e)\varphi Y\left(\nu_K+d_p\sqrt{\frac{2k_p}{3\pi}}\right),$$
$$\xi=\frac{4}{3}(1+e)\varphi Yd_p\sqrt{\frac{2k_p}{3\pi}}, \tag{24}$$
$$\Lambda_C=2(1+e)\varphi Y\left(\frac{3}{5}\Lambda_K+d_p\sqrt{\frac{2k_p}{3\pi}}\right).$$

Total stresses and the pulsation energy diffusion flux in the dispersed phase, in accordance with Eqs. (19), (20), (22), and (23), are defined by the expressions

$$P_{ij} = \left(P_p - \varphi\xi \frac{\partial V_k}{\partial x_k} \right) \delta_{ij} - \varphi\nu_p S_{ij}, \tag{25}$$

$$q_i = -\varphi\Lambda_p \frac{\partial k_p}{\partial x_i}, \tag{26}$$

where the pressure P_p, the shear viscosity coefficient ν_p, and the pulsation energy diffusion coefficient Λ_p of particles, in view of Eq. (24), are equal to

$$P_p = \frac{2}{3} \varphi k_p [1 + 2(1+e)\varphi Y], \tag{27}$$

$$\nu_p = \nu_K + \nu_C = \left[1 + \frac{4}{5}(1+e)\varphi Y \right] \nu_K + \frac{3}{5}\xi, \tag{28}$$

$$\Lambda_p = \Lambda_K + \Lambda_C = \left[1 + \frac{6}{5}(1+e)\varphi Y \right] \Lambda_K + \frac{3}{2}\xi. \tag{29}$$

Expressions (12), (25), and (26) complete the system of movement Eqs. (2)-(4), describing the mass, momentum, and pulsation energy transfer in the dispersed phase. Let us examine the limiting relations for the transfer coefficients ν_p and Λ_p in rarefied and concentrated dispersed systems.

For $\varphi << 1$ Eqs. (28) and (29), with consideration of the terms of the first order by φ ($Y = 1 + O(\varphi)$) yield

$$\nu_p = \frac{\tau k_p}{3} + \frac{2(1+e)}{15} \tau k_p \left[(3e-1) - \right.$$
$$\left. - \frac{4(3-e)\tau k_p^{1/2}}{d_p} \sqrt{\frac{3}{2\pi}} + \frac{6 d_p}{\tau k_p^{1/2}} \sqrt{\frac{2}{3\pi}} \right] \varphi + O(\varphi^2),$$
$$\Lambda_p = \frac{10}{27}\tau k_p + \frac{10(1+e)}{27} \tau k_p \left[\frac{3}{5}(1+e)(2e-1) - \right.$$
$$\left. - \frac{4(49-33e)\tau k_p^{1/2}}{45 d_p} \sqrt{\frac{3}{2\pi}} + \frac{27}{5} \sqrt{\frac{2}{3\pi}} \frac{d_p}{\tau k_p^{1/2}} \right] \varphi + O(\varphi^2).$$

It is seen from these relations, that depending on the value of the parameter $\Delta = d_p/\tau k_p^{1/2}$ the influence of the flow constraint for small volume concentrations φ may lead to both increase and decrease in particle transfer coefficients.

The possibility of nonmonotonic change in the transfer coefficients of the particle concentration for small values of the parameter Δ is confirmed by the dependences, constructed according to Eqs. (28) and (29) (see Fig. 1 below). In concentrated systems, in contrast to rarefied ones, the growth of the transfer coefficients takes place along with the growth of the particle concentration for all values of the parameter Δ.

Although the obtained results, based on the presentation of the collision integral with the help of the Enskog expansion are valid, strictly speaking, for the rarefied dispersed medium, it can be expected that for successful approximation of the dependence $Y(\varphi)$ they can be used in concentrated systems as well. Thus, for a high volume concentration of particles the dependence $Y(\varphi)$ can be found from the condition that expression (27) for pressure coincides with the equation of state for the dense gas of elastic spherical particles [7]

$$P_p = \frac{2\varphi k_p}{3[1 - (\varphi/\varphi_m)^{1/3}]},$$

whence

$$Y = \frac{1}{4\varphi_m^{1/3}\varphi^{2/3}[1 - (\varphi/\varphi_m)^{1/3}]}. \tag{30}$$

Dependence (30) is not valid for rarefied dispersed system (for $\varphi \to 0$) and, therefore, it can be used for sufficiently concentrated systems only. Taking into account (30), expression (12) for the pulsation energy dissipation becomes close to the one obtained in [9].

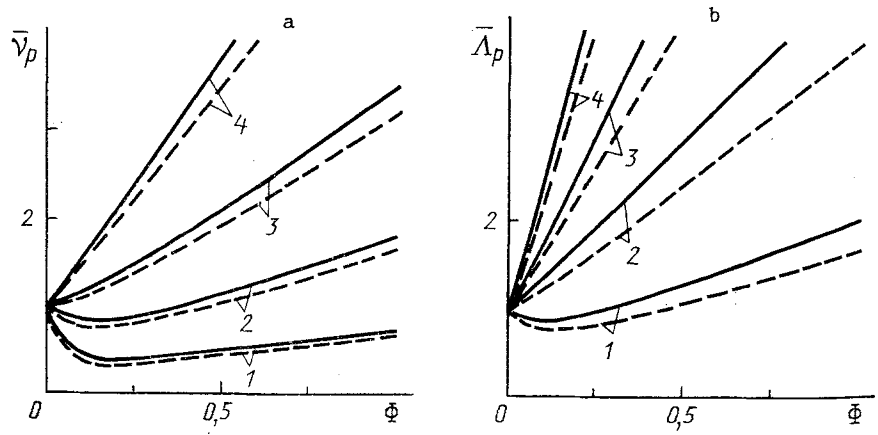

Fig. 1. The shear viscosity coefficient (a) and the pulsation energy diffusion coefficient (b): 1) Δ = 0.2; 2) 0.5; 3, 1; 4, 2; unbroken curves) e = 1.0; dashed curves) 0.9.

For the approximation of $Y(\varphi)$ within the entire range of φ, i.e., for $0 < \varphi < \varphi_m$, dependence [12] can be used

$$Y = [1 - (\varphi/\varphi_m)^{1/3}]^{-1},$$

satisfying the condition $Y \to 1$ for $\varphi \to 0$ and having the same asymptotic character for $\varphi \to \varphi_m$ as (30).

NOTATION

t, time; x_i, coordinate; v_i, v_i', and V_i, actual, pulsation and mean particle velocities; U_i, mean velocity of the carrier flow; τ, time of dynamic particle relaxation; T, time integral turbulence scale; f_i, external mass force and interphase interaction force; φ, dispersed phase volume concentration; d_p, particle diameter; $D = 2Tk/3$, turbulent diffusion coefficient of inertialess impurity; k, turbulent energy of carrier flow; $k_p = \langle v_k' v_k' \rangle /2$, particle pulsation energy; e, coefficient of momentum recovery during a collision; ν_{p0}, Λ_{p0}, transfer coefficients for $\varphi = 0$; $\bar{\nu}_p = \nu_p/\nu_{p0}$; $\bar{\Lambda}/\Lambda_{p0}$ and $\Phi = \varphi Y$.

LITERATURE CITED

1. I. V. Derevich and L. I. Zaichik, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 5, 96-104 (1988).
2. I. V. Derevich and L. I. Zaichik, Prikl. Mat. Mekh., **54**, No. 5, 767-774 (1990).
3. S. Chandrasekhar, Stochastic Problems in Physics and Astronomy [in Russian], Moscow (1947).
4. V. M. Voloshchuk, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, **35**, No. 2, 155-162 (1970).
5. Yu. A. Buevich, Prikl. Mat. Mekh. **35**, No. 3, 464-481 (1971).
6. I. V. Derevich and V. M. Eroshenko, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 2, 69-78 (1990).
7. J. Hirshfelder, Ch. Kurtis, and R. Bird, Molecular Theory of Gases and Liquids [Russian translation], Moscow (1961).
8. M. I. Kogan, Rarefied Gas Dynamics [in Russian], Moscow (1967).
9. M. A. Gol'dshtik and B. N. Kozlov, Zh. Prikl. Mekh. Tekh. Fiz., No. 4, 67-77 (1973).
10. C. K. K. Kun, S. B. Savage, D. J. Jeffrey, and N. J. Chepurniy, J. Fluid Mech., **140**, 223-256 (1984).
11. J. Ding and D. Gidaspow, AIChE J., **36**, No. 4, 523-538 (1990).
12. S. Ogawa, A. Umemura, and N. Oshima, J. Appl. Math. Phys., **31**, 483-493 (1980).